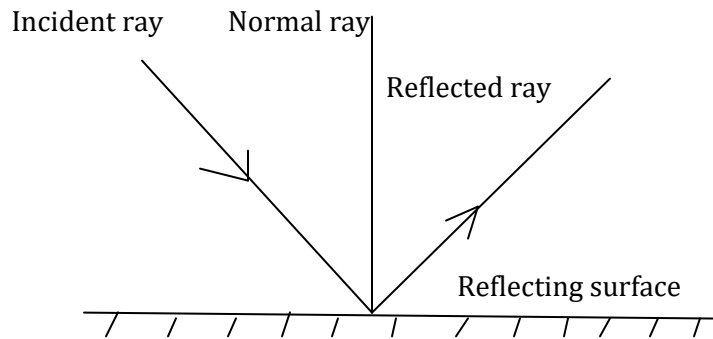


# OPTICS

## REFLECTION

This is the bouncing back of light after striking a reflecting surface.



## LAWS OF REFLECTION

1. The angle of incidence is equal to the angle of reflection ( $i=r$ ).
2. The incident ray, reflected ray and the normal lie in the same plane at the point of incidence.

## TYPES OF REFLECTION

There are two types of reflection, namely:

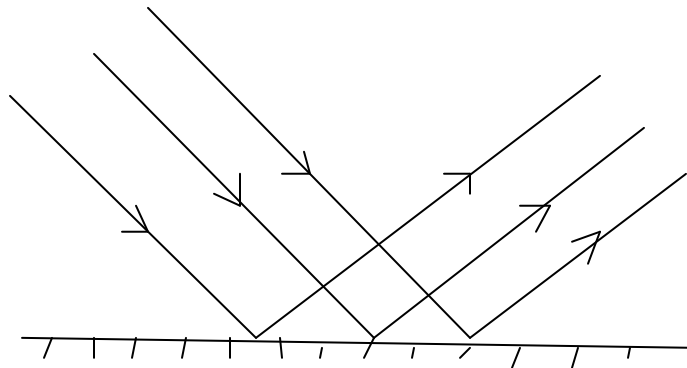
- Regular reflection
- Diffuse/Irregular reflection

## REGULAR REFLECTION

Characteristics of regular reflection

- ❖ The reflecting surface is smooth
- ❖ A parallel incident beam is reflected as a parallel reflected beam.
- ❖ The laws of reflection are obeyed.

## Diagram



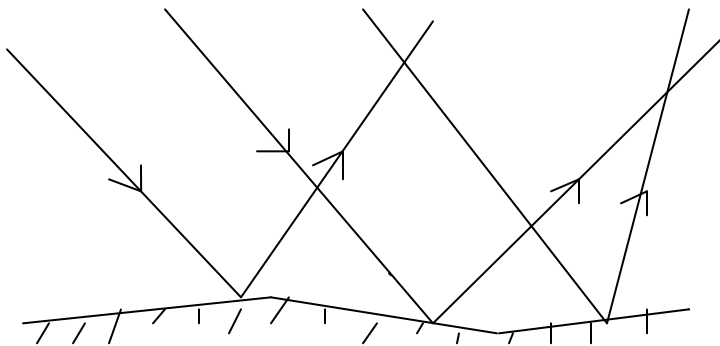
## DIFFUSE/IRREGULAR REFLECTION

Characteristics of diffuse reflection.

The reflection takes place at a rough surface.

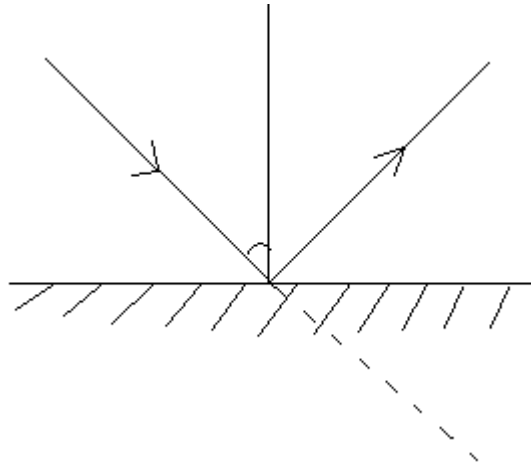
Parallel incident beam is not reflected as a parallel beam.

The laws of reflection are not obeyed.



## DEVIATION OF LIGHT BY A PLANE SURFACE

Deviation refers to the change in the direction of a light ray.



g- Glancing angle

$\alpha$ - Angle of deviation

i- Angle of incidence

r- Angle of reflection

Using  $i = r$

$$AON = NOC$$

$$AOX = COY = g$$

$$BOY = AOX = g(\text{alternate angles})$$

$$\alpha = COY + BOY$$

$$\alpha = g + g$$

$$\alpha = 2g.$$

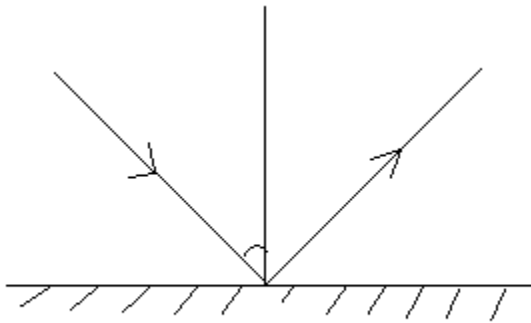
### Note:

A ray is the direction taken by light energy.

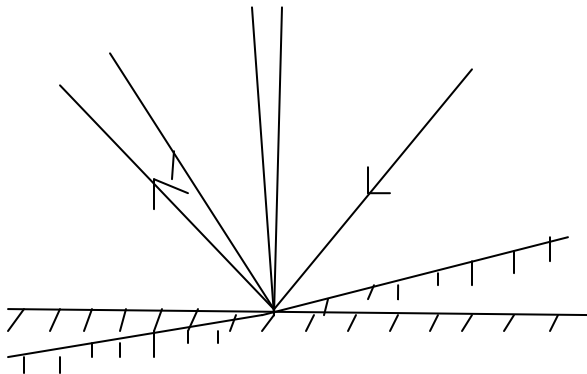
A beam is a collection of light rays.

## ROTATION OF A MIRROR

Consider the ray IO incident on a plane mirror.



If the mirror is rotated through an angle  $\theta$ , then the reflected ray will be rotated through an angle  $2\theta$ . The incident ray IO is kept fixed. MM is rotated to  $M_1M_1$  through an angle  $\theta$ .



$$\text{Angle } ION_1 = i + \theta = \text{Angle } R_1ON_1$$

The angle of rotation of the reflected ray = angle  $R_1OR$

$$R_1OR = R_1ON - RON$$

$$R_1OR = R_1ON_1 + N_1ON - RON$$

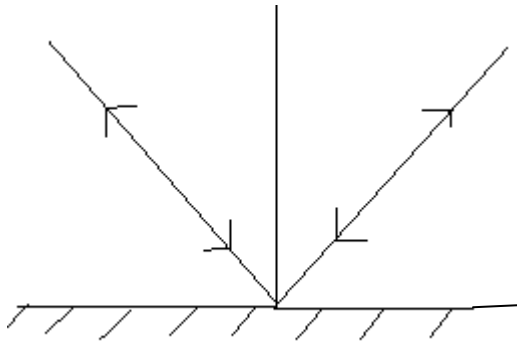
$$R_1OR = i + \theta + \theta - i$$

$$R_1OR = 2\theta$$

Hence if the mirror is rotated through an angle  $\theta$  while keeping the position of the incident ray constant, the angle of rotation of the reflected ray is  $2\theta$ .

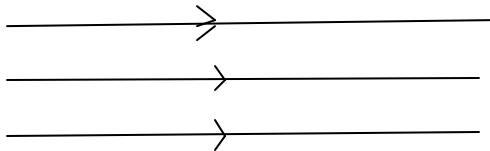
### PRINCIPLE OF REVERSIBILITY OF LIGHT

The principle states that if the direction of a light ray is reversed, it travels along its original path.

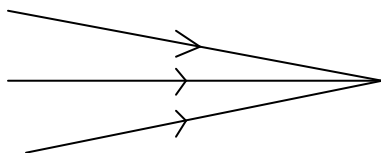


### TYPES OF BEAMS

1. Parallel beam e. g Search light



2. Convergent beam e. g Latern

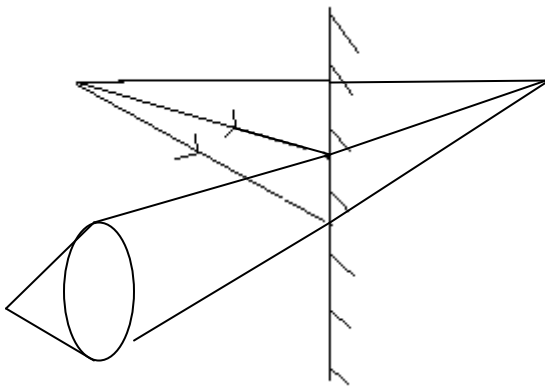


3. Divergent beam e. g Lamp



### **IMAGES FORMED BY A PLANE MIRROR**

Consider a point object O as shown below.



Ray OA strikes the mirror surface normally and it is reflected along the same path.

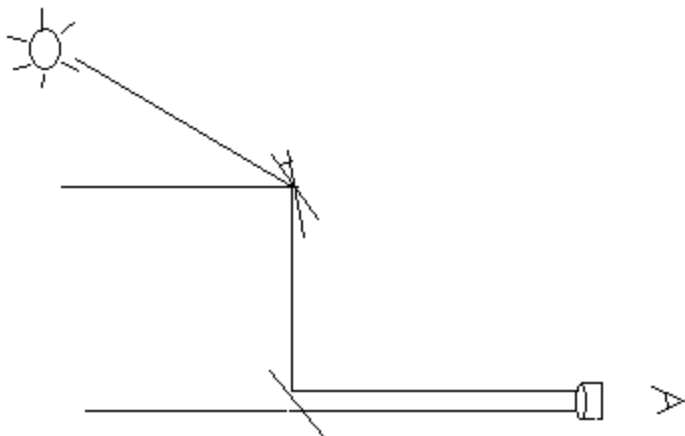
Ray OD is reflected along DE: where E is the eye position

When OA is produced, it meets ED produced at I which is the image of O.

The eye sees the image in the direction in which light enters it.

### **PRINCIPLE OF THE SEXTANT**

The set up is used to measure the angle of elevation of the sun and stars.



Mirror B is silvered on a vertical half. Looking through T, the mirror is turned about the horizontal axis until the view  $H^1$  of the horizon is seen directly through the unsilvered half of B and also the view of H seen by successive reflection at O and the silvered part of B are coincident.

O is then parallel to B in the position  $M_1$ . O is now rotated to position  $M_2$  until the image of the sun S is seen on the horizon  $H^1$ .

The angle of rotation  $\theta$ , of the mirror at O is noted, the elevation SOH of the sun will be equal to  $2\theta$ .

### **SUCCESSIVE REFLECTION AT TWO PLANE MIRRORS**

If two mirrors are inclined at an angle  $\theta$ , the net deviation after successive reflection in the two mirrors is twice the angle of inclination.

### **REFLECTION AT CURVED SURFACES**

Terms used in curved mirrors. There are two types of mirrors i.e concave and convex

- i) Point C is called the centre of curvature: This is the point where the mirror forms a 'cut' on the principal axis.
- ii) The line CP which joins the centre of curvature and the point P is called the principal axis.
- iii) The midpoint, P is called the pole of the mirror.
- iv) Marginal rays are rays that are parallel to and far away from the principal axis.
- v) Paraxial rays are rays which are parallel to and close to the principal axis.
- vi) Distance CP is called the centre of curvature

vii) Principal focus,  $F$  is a point on the principal axis through which all paraxial rays appear to be reflected from for a convex mirror or from which rays are reflected on a concave mirror.

viii) Focal length,  $f$  is the distance between the principal focus and the pole of the mirror.

ix) Radius of curvature,  $r$  is the distance between the centre of curvature and the pole of the mirror.

x) For a wide beam of light incident on a wide mirror, the reflected rays do not pass through a single point on the principle axis of the spherical mirror. The locus of points of intersection of such a beam of light reflected from a wide mirror is called a caustic surface.

### USES OF CONCAVE MIRROR

1. Used in reflector telescopes
2. Used in projectors
3. Used as a dressing mirror

### USES OF CONVEX MIRRORS

1. Used as driving mirrors
2. Used for surveillance in supermarkets

### Suitability

- . Has a wide field of view
- . Forms erect images

### RELATIONSHIP BETWEEN $r$ and $f$

To derive the relationship between  $r$  and  $f$ , consider the diagram below for a concave mirror

Consider the triangle  $CMP$

Since  $M$  is close to  $P$ ,  $\theta$  is small and  $\tan \theta \approx \theta$

$$\text{Thus } \theta \approx \frac{MP}{CP} \dots\dots\dots(i)$$

Consider triangle  $FMP$



$$\tan 2\theta = \frac{MP}{FP} \text{ but } \tan 2\theta \approx 2\theta$$

$$2\theta = \frac{MP}{FP} \dots\dots\dots(ii)$$

Solving (i) and (ii)

$$2 \frac{MP}{CP} = \frac{MP}{FP}$$

$$CP = 2FP; FP = f, CP = r$$

$$\text{Hence } r = 2f$$

Assignment Derive the relationship for a convex mirror

### IMAGES IN SPHERICAL MIRRORS

These depend on the distance of the object from the mirror. The image may be real or virtual, diminished or magnified, inverted or upright. The position of the image may be determined by ray diagram or by calculation using the mirror formula

(a) Ray diagrams

Take a small object at the principal axis so that only paraxial rays are considered. To construct the image, two of the following 3 rays are considered from the top of the object

1 A ray parallel to the principal axis which after reflection passes through the principal focus or appears to diverge from it.

2 A ray through the center of curvature which strikes the mirror normally and is reflected back along the same path.

3 A ray through the principal focus which is reflected parallel to the principal axis. The following are the constructional ray diagrams for a concave mirror

(i) Object beyond C

Image between C and F, real, inverted and diminished

(ii) Object at C

Image is at C, real, inverted and the same size as the object

(iii) Object between C and F

Image beyond C, real, inverted and magnified

iv) Object at F

Image is formed at infinity, real, inverted and magnified

v) Object between F and P

Image beyond the mirror, virtual, upright and magnified

For a convex mirror, no matter where the object is, the image is always virtual, upright and diminished.

### MIRROR FORMULA

Consider the following diagrams

Concave mirror

Convex mirror

O is the point object on the principal axis

OM is the incident ray

OP is a ray which strikes the mirror normally and is reflected along PO

MI (reflected ray) meets PO at I

Concave mirror

Triangle CMO

$$\beta = \alpha + \theta$$
$$\Rightarrow \theta = \beta - \alpha$$

Convex mirror

Triangle CMO

$$\theta = \alpha + \beta$$

Triangle CMI

Tiangle CMI

$$\begin{aligned}\gamma &= \theta + \beta \\ \text{Thus } \theta &= \gamma - \beta \\ \gamma - \beta &= \beta - \alpha \\ 2\beta &= \alpha + \gamma \dots\dots\dots(i)\end{aligned}$$

$$\begin{aligned}\gamma &= \theta + \beta \\ \Rightarrow \theta &= \gamma - \beta \\ \gamma - \beta &= \alpha + \beta \\ 2\beta &= \gamma - \alpha \dots\dots\dots(ii)\end{aligned}$$

The mirrors have small apertures and M is close to P. The angles  $\alpha, \beta, \gamma$  are small and measured in radians, hence  $\tan \alpha \approx \alpha$ ,  $\tan \beta \approx \beta$  and  $\tan \gamma \approx \gamma$ .

$$\text{But } \tan \alpha = \frac{MP}{OP} = \frac{MP}{u}, \tan \beta = \frac{MP}{CP} = \frac{MP}{r} \text{ and } \tan \gamma = \frac{MP}{IP} = \frac{MP}{v}$$

Where  $u$ - Object distance,  $v$ - Image distance and  $r$ - Radius of curvature

Concave mirror

$$\frac{2MP}{r} = \frac{MP}{v} + \frac{MP}{u}$$

Since  $r = 2f$

$$\frac{2}{2f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

Convex mirror

$$\frac{2MP}{r} = \frac{MP}{v} - \frac{MP}{u}$$

$$\frac{2}{2f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

### **MAGNIFICATION, $m$**

This is defined as the ratio of the height of the image to the height of the object i. e

$$\text{Magnification} = \frac{\text{height of image}}{\text{height of object}}$$

From the laws of reflection, angle APO = angle IPB

Triangle APO and IPB are similar

$$M = \frac{\text{height of image}}{\text{height of object}} = \frac{IB}{OA} = \frac{IP}{OP} = \frac{v}{u}$$

$$m = \frac{v}{u}$$

From  $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$  multiplying through by  $v$  yields

$$\frac{v}{f} = \frac{v}{v} + \frac{v}{u}$$

$$\frac{v}{f} = 1 + m$$

$$m = \frac{v}{f} - 1$$

From  $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$  multiplying through by  $u$  yields

$$\frac{u}{f} = \frac{u}{v} + \frac{u}{u}$$

$$\frac{u}{f} = \frac{1}{m} + 1$$

$$\text{Thus } m = \frac{f}{u - f}$$

Examples

1. An object is placed 15cm from

i) .concave mirror

ii)a convex mirror of radius of curvature 20cm, calculate the image position and the magnification in each case.

Solution by Geof

$$u = 15\text{cm}, f = 10\text{cm}$$

Concave mirror

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{15}$$

$$v = 15\text{cm}$$

$$\text{Magnification} = \frac{30}{15} = 2$$

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{-1}{v} = \frac{-1}{-10} + \frac{1}{15}$$

$$v = -6\text{cm}$$

$$\text{Magnification} = \frac{6}{15} = 0.4$$

2. An erect image 3 times the size of the object is obtained with a concave mirror of radius of curvature 36cm. Find the object distance

Solutions by Geof

Method 1

$$\text{From } r = 2f, f = \frac{36}{2} = 18\text{cm}$$

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$m = \frac{v}{f} - 1$$

$$3 = \frac{v}{18} - 1$$

$$v = 72\text{cm}$$

$$\frac{1}{18} = \frac{1}{72} + \frac{1}{u}$$

$$u = 24\text{cm}$$

Method 2

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{18} = \frac{1}{u} + \frac{1}{3u}$$

$$u = 24\text{cm}$$

Method 3

$$\text{Thus } m = \frac{f}{u - f}$$

$$u = f \left( \frac{1}{m} + 1 \right)$$

$$= 18 \left( \frac{1}{3} + 1 \right)$$

$$= 24\text{cm}$$

## DETERMINATION OF $f$ AND $r$

### CONCAVE MIRROR

#### Method I: No parallax method

- Place the concave mirror on a table with its reflecting surface facing upwards
- Move an optical pin up and down along the line of the principal axis till it coincides with its own image as shown below.
- Measure and record the distance  $r$  from the mirror to the object using a meter ruler clamped next to a mirror.
- Obtain  $f$  from  $f = \frac{r}{2}$

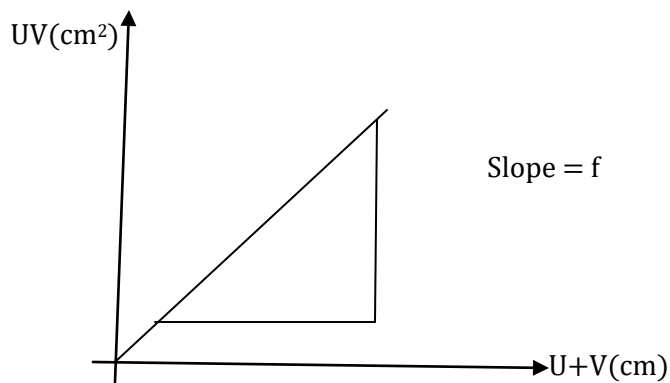
#### Method II

The method operates by placing the object at distances  $u$  from the pole of the mirror and the corresponding values of the image distances are recorded. A set of results may be entered in a table as indicated below.

TABLE OF RESULTS

| U(cm) | V(cm) | UV(cm <sup>2</sup> ) | U+V(cm) |
|-------|-------|----------------------|---------|
|       |       |                      |         |

A graph of  $UV$  against  $U+V$  is plotted



### CONVEX MIRROR

#### METHOD I: Plane mirror method

a) An object pin is placed in front of a convex mirror and a virtual image  $I$  is formed. A small plane mirror  $M$  is then placed between the convex mirror and the object is moved until the image  $I^1$  of the lower part of the object coincides with the upper part of the object in the convex mirror.

Distances  $OP$  and  $MP$  are then measured

Now  $OM = MI^1$  for the plane mirror  $M$  and  $PI = MI^1 - MP$

But  $PI = v$  - Image distance

$OP = u$  - Object distance

$$f = \frac{uv}{u+v} \text{ and } r = 2f$$

#### Method II

Auxiliary converging lens method will be discussed after lenses.

### REFRACTIVE INDEX OF A LIQUID USING A CONCAVE MIRROR

a) A concave mirror is placed on a bench facing upwards.

b) An optical pin is clamped horizontally in a retort stand so that its apex is along the principal axis of the mirror.

c) The height of the pin above the mirror is adjusted until it coincides with its image and there is no parallax between the image and the pin.

d) The height, R of the pin above the pole of the mirror is noted. A little quantity of the liquid whose refractive index is required is placed in the mirror.

e) The height of the pin is adjusted until the pin again coincides with its image, the height, h of the pin above the liquid is measured.

f) The refractive index, n of the liquid can be calculated from as follows

$$n = \frac{\sin i}{\sin r}$$

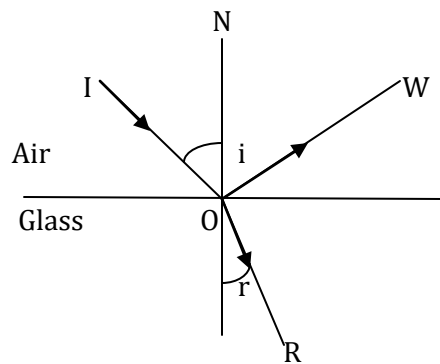
$$n = \frac{\frac{NM}{IM}}{\frac{NM}{CM}} = \frac{CM}{IM}$$

By observation directly above P,  $CM \approx CN$  and  $IM \approx IN$

Hence  $n = \frac{CN}{IN} = \frac{R-d}{n}$ ; where d is the depth of the liquid

## REFRACTION

This is the bending of light rays as it travels from one medium to another e. g air-glass media



OI – Incident ray

OR – Reflected ray

i – Angle of incidence

r – Angle of refraction

OW - Weak reflected ray

ON – Normal



Some small portion of the light energy may be reflected back into the first medium but this reflected ray is much weaker as compared to the refracted ray.

### LAWS OF REFRACTION

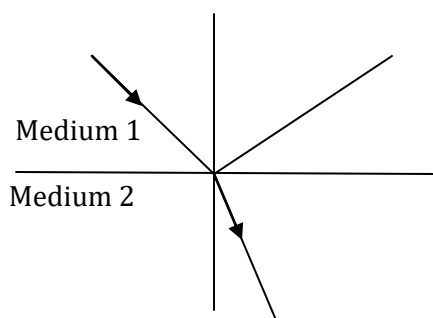
1. The refracted ray is in the same plane as the incident ray and the normal to the medium at the point of incidence but in different medium.
2. For a given pair of media, the ratio of sine of the angle of incidence to the sine of the angle of refraction is a constant i.e  $\frac{\sin i}{\sin r} = \text{constant}$ ; This is called Snell's law

### REFRACTIVE INDEX

The  $\frac{\sin i}{\sin r}$  is called the refractive index for light travelling from the first to the second medium.

If the medium containing the incident ray is denoted by 1 and that containing the refracted ray by 2, then the refractive index is denoted as  ${}_1n_2$

$${}_1n_2 = \frac{\sin i}{\sin r}$$

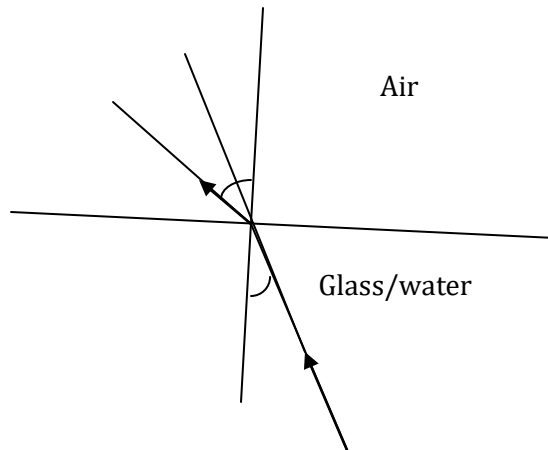


The magnitude of the refractive index depends on the color of light and is usually stated for yellow light. If the first medium is air/vacuum, we usually refer to the absolute refractive index of medium 2 and is denoted as  ${}_an_2$  or  $n_2$ .

The absolute refractive index for crown glass = 1.50 and that for water = 1.33. The greater the refractive index of a medium, the greater the change in the direction suffered by light travelling from air to the medium e. g refraction for air-glass boundary has a bigger change in direction rather than refraction in air-water boundary. In both cases however, the refracted ray is bent towards the normal and light is said to be travelling to an optically denser medium.

A ray of light travelling from glass or water to air would thus be bent away from the normal.

Refractive index of air = 1.00



Refractive index can also be defined in terms of velocity of light in the two medium as follows

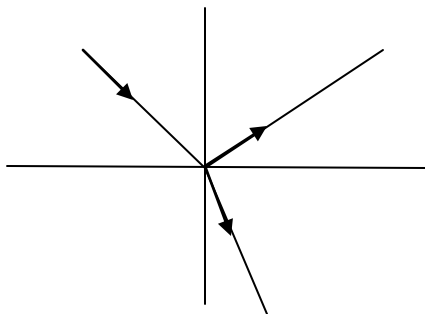
$${}_1n_2 = \frac{\text{velocity of light in medium 1}}{\text{velocity of light in medium 2}}$$

$$\text{For absolute refractive index} = \frac{\text{velocity of light in air / vacuum}}{\text{velocity of light in the required medium}} = \frac{c}{v}$$

Hence refractive index may also be defined as the ratio of the speed of light in air to the speed of light in the second medium.

#### REFRACTIVE INDEX RELATIONS

1. Air-glass or air-water media



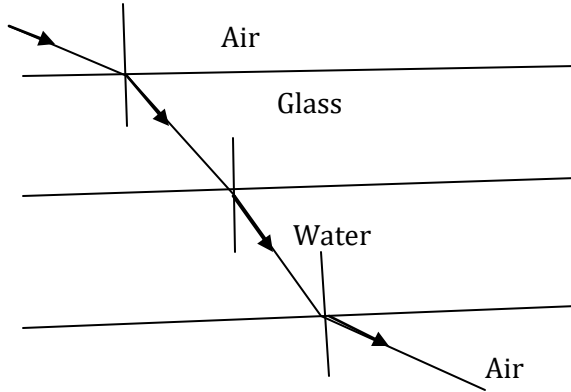
$${}_1n_2 = \frac{\sin i}{\sin r}$$

From the principle of reversibility of light, a ray RO would be refracted through OI.

$${}_2n_1 = \frac{\sin r}{\sin i}$$

$${}_1n_2 = 1/{}_2n_1$$

## 2. Air-glass-water-air media



For parallel media, the emergent ray although laterally displaced is parallel to the incident ray and hence the angle of incidence in the air-glass boundary is equal to the angle of refraction in the water-air boundary.

$${}_1n_2 = \frac{\sin i_a}{\sin i_g} \quad {}_2n_3 = \frac{\sin i_g}{\sin i_w} \quad {}_3n_1 = \frac{\sin i_w}{\sin i_a}$$

$${}_1n_2 \times {}_2n_3 \times {}_3n_1 = \frac{\sin i_a}{\sin i_g} \times \frac{\sin i_g}{\sin i_w} \times \frac{\sin i_w}{\sin i_a} = 1$$

Find the angle of refraction of water-glass incidence of angle  $50^\circ$ . ( $n_w=1.33$ ,  $n_g=1.50$ )

$$n_w \sin i_w = n_g \sin i_g$$

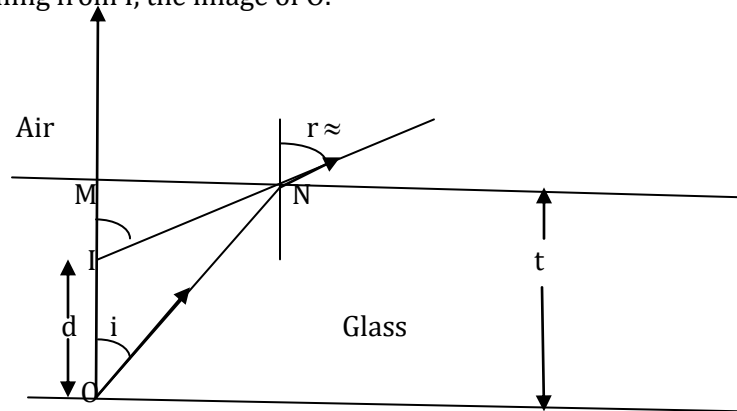
$$\sin i_g = n_w \sin i_w / n_g$$

$$= \frac{1.33 \sin 50}{1.5}$$

$$i_g = 42.8^\circ$$

## APPARENT DEPTH

The bottom of a clear pool of water appears nearer to the surface when viewed from above and also an object under water appears to be nearer to the surface than its actual position. These are due to refraction of light. The diagram below shows rays from a point object under a glass block. Rays from a point object are bent away from the normal at the glass-air boundary and they appear to be coming from I, the image of O.



$$n_g \sin i = n_a \sin r$$

$$n \sin i = \sin r; \text{ since } n_a = 1$$

$$n = \frac{\sin r}{\sin i}$$

$$\text{but } \sin i = \frac{MN}{NO} \quad \text{and } \sin r = \frac{MN}{IN}$$

Since N is close to M,

$$ON \approx OM \text{ and } IN \approx IM$$

$$\text{Then } \sin i = \frac{MN}{OM} \text{ and } \sin r = \frac{MN}{MI}$$

$$n = \frac{MN}{IM} * \frac{OM}{MN} = \frac{OM}{IM}$$

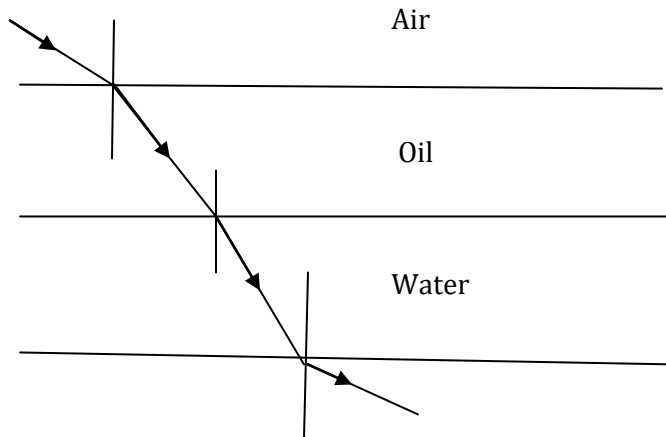
$$= \frac{\text{Real depth}}{\text{Apparent depth}} \quad n = \frac{t}{t-d}$$

$$nt - nd = t \quad \Rightarrow \quad d = t \left( 1 - \frac{1}{n} \right)$$

Examples.

1. An oil film of refractive index 1.40 floats on water. Calculate the angle of refraction in the water for the ray of light that strikes the oil surface at  $35^\circ$  to the normal.

Solution



Applying snell's law

$$n_a \sin i_a = n_o \sin i_o = n_w \sin i_w$$

$$n_w \sin i_w = n_o \sin i_o$$

$$\sin i_w = \frac{n_o \sin i_o}{n_w} = \frac{\sin 35}{1.33}$$

$$i_w = 25.6^\circ$$

2. Calculate the real depth of a swimming pool if the apparent depth is 1.2m.

Solution

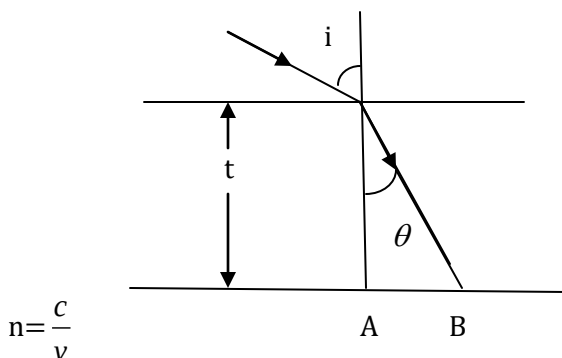
$$n = \frac{\text{Real depth}}{\text{apparent depth}}$$

$$1.33 = \frac{R.D}{1.2}$$

$$\text{real depth} = 1.59$$

3. Monochromatic light incident on a block of material placed in a vacuum is refracted through angle  $\theta$ . If the block has refractive index  $n$  and the thickness,  $t$ , show that light takes a time  $\frac{nt \sec \theta}{c}$  to emerge from a block;  $c$  is the speed of light in a vacuum.

Solution.



$$\text{let } t_1 \text{ be the time, } t_1 = \frac{\text{distance}}{\text{speed}} = \frac{OB}{V}$$

$$OB = \frac{t_1}{\cos \theta},$$

$$OB = t \sec \theta$$

$$t_1 = \frac{t \sec \theta}{V}; \text{ but } \frac{1}{V} = \frac{n}{c}$$

$$\text{thus } t_1 = \frac{nt \sec \theta}{c}, \text{ hence shown.}$$

## TOTAL INTERNAL REFLECTION.

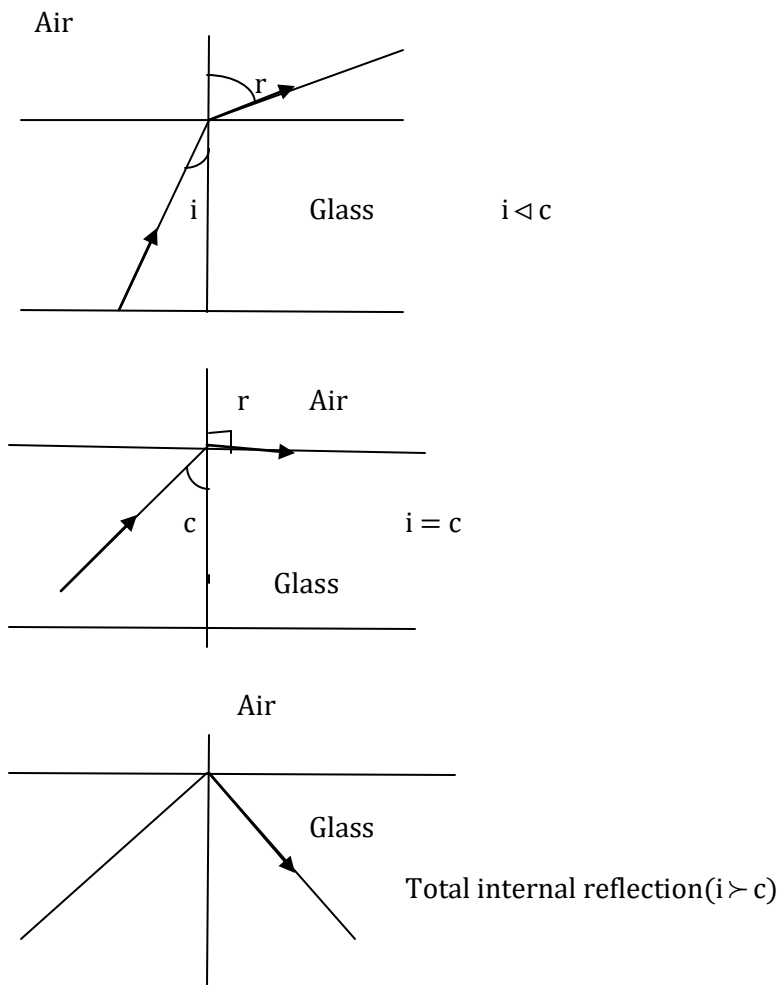
Definition:

Total internal reflection is the reflection of light back into the same medium for a ray of light travelling from a denser medium to a less dense medium.

When light travels from a material such as glass into a material of lower refractive index e.g air, experiment shows that there is a maximum angle of incidence that will give a refracted ray i.e the angle of incidence for which the angle of refraction is  $90^\circ$ .

The refracted ray moves along the surface of the glass. The angle of incidence in the denser medium corresponding to the angle of refraction of  $90^\circ$  is called the critical angle.

At this point, the internally reflected ray is still weak but as  $c$  is slightly increased, the reflected ray suddenly becomes brighter and the refracted ray suddenly disappears i.e the light is reflected back into the optically denser medium. This is called total internal reflection since all the incident ray is reflected back into the same medium.



Applying snell's law

$$n \sin i = \text{constant}$$

$$n_g \sin c = n_a \sin 90^\circ \text{ ; at critical angle}$$

$$n_g = \frac{1}{\sin c}$$

for crown glass,  $n_g = 1.50$

$$\sin c = \frac{i}{n_g} = \frac{1}{1.50}$$

$$c = 41.8^\circ$$

so, when the incident angle exceeds  $42^\circ$ , total internal reflection occurs for crown glass.

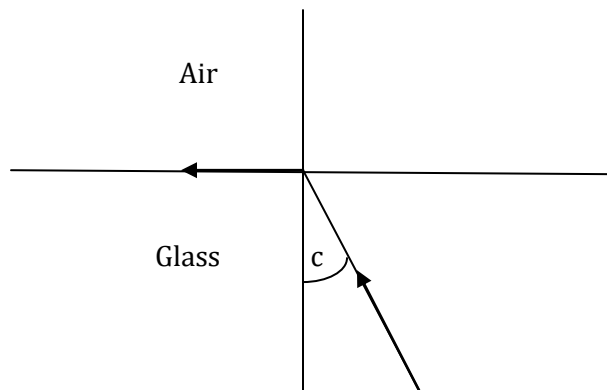
N.B: Total internal reflection does not occur for light travelling from a less dense medium to a denser one since there will always be refraction.

Examples.

1. Calculate the critical angle for a light ray that is incident on a water glass boundary.

Solution.

$$n_w = 1.33 \text{ and } n_g = 1.50$$



Applying Snell's law

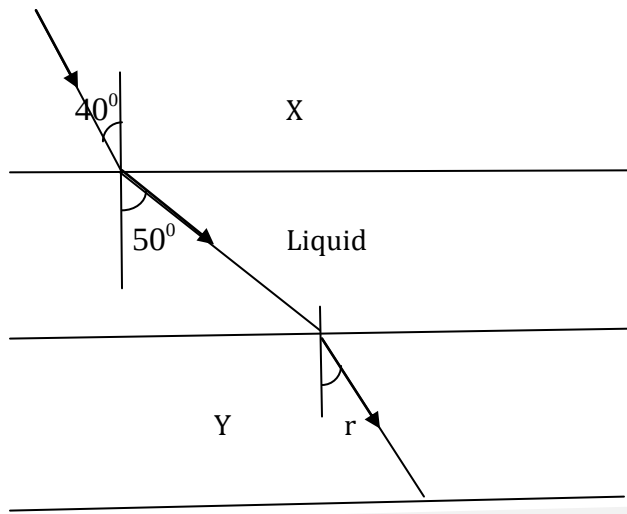
$$n_g \sin C = n_w \sin 90$$

$$\sin C = \frac{n_w}{n_g} = \frac{1.33}{1.50} = 0.8866$$

$$C = \sin^{-1} 0.8866 = 62.4^\circ$$

2. The figure below shows a layer of liquid in between two transparent plates X and Y of refractive indices 1.54 and 1.44 respectively.





A ray of light making an angle of  $40^\circ$  to the normal is refracted as indicated. Find the,

i) Refractive index of the liquid'

ii) Angle of refraction,  $r$  in medium Y.

iii) Minimum angle of incidence in X for which the light will not emerge from Y.

Solution;

i) Using snell's law,

$$n_1 \sin i_1 = n_2 \sin i_2$$

$$n_l = \frac{n_1 \sin i_1}{\sin i_2}$$

$$n_l = \frac{1.5 \sin 40}{\sin 50}$$

$$n_l = 1.29$$

$$\text{ii) } n_1 \sin i_1 = n_2 \sin i_2 = n_3 \sin i_3$$

$$n_1 \sin i_1 = n_3 \sin i_3$$

$$\sin i_3 = \frac{1.54 \sin 40}{1.44}$$

$$\sin i_3 = 0.6874$$

$$i_3 = 43.4^\circ$$

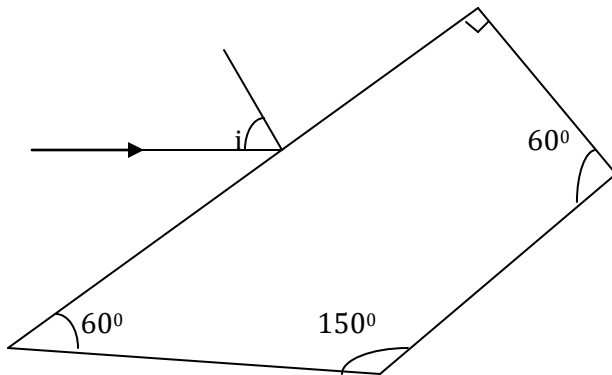
iii)  $i_{\text{mm}}$  where  $r = 90^\circ$

$$n_1 \sin i_{\text{mm}} = n_2 \sin 90^\circ$$

$$\sin i_{\text{mm}} = \frac{1}{1.54}$$

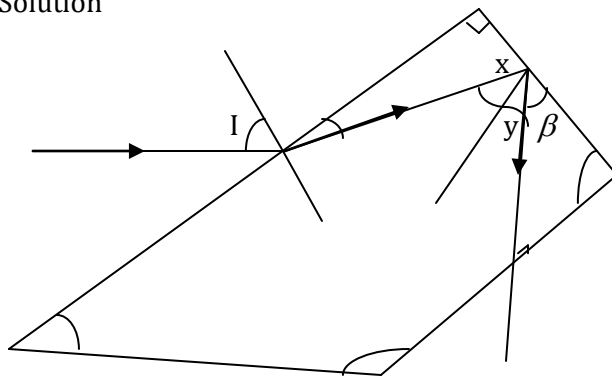
$$i_{\text{mm}} = 40.5^\circ$$

3. A ray of light is incident on a face AD on a glass block below. The refractive index of the glass block is 1.52.



If the ray emerges normally through the face BC after total internal reflection. Calculate the value of angle  $i$ .

Solution'



$$\beta = 90 - 60 = 30^\circ$$

$$y = 90 - \beta = 60^\circ$$

$$x = \beta = 30^\circ$$

$$k = 90 - x = 90 - 30$$

$$k = 60^\circ$$

$$r = 90 - 60 = 30^\circ$$

using snell's law,

$$n_1 \sin i_1 = n_2 \sin i_2$$

$$\sin i_2 = \frac{n_2 \sin i_2}{n_1}$$

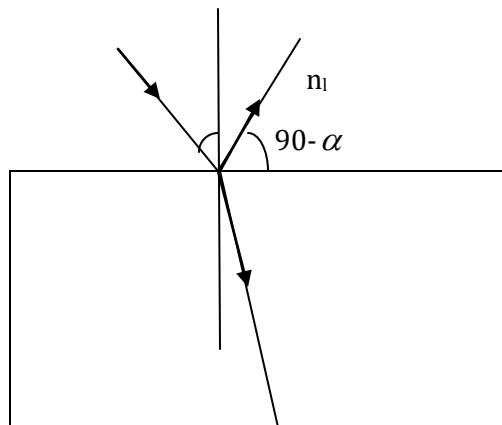
$$\sin i_1 = 1.45 \sin 30$$

$$\therefore i_1 = 49.5^\circ$$

4.a) A glass block of refractive index,  $n_g$  is immersed in a liquid of  $n_l$ . a ray of light is reflected and refracted at the interface such that the angle between the reflected ray and the refracted ray is  $90^\circ$ . Show that  $n_g = \tan \alpha$ . Where  $\alpha$  is the angle of incidence from liquid to glass.

b) when the procedure above is repeated with the liquid removed, the angle of incidence increases by  $8^\circ$ . Given that  $n_l = 1.33$ , find  $n_g$ .

Solution.



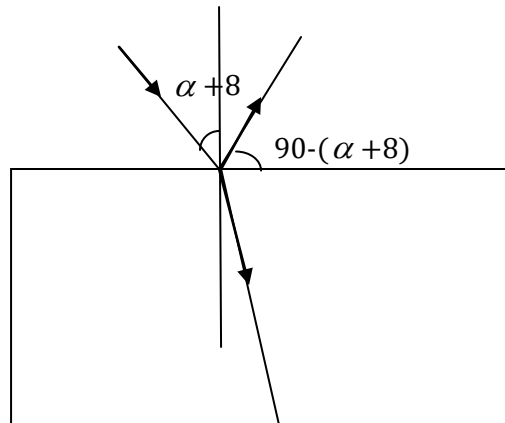
$$n_l \sin \alpha = n_g \sin(90 - \alpha)$$

$$n_l = n_g \sin\{\sin 90 \cos \alpha - \sin \alpha \cos 90\}$$

$$n_l \sin \alpha = n_g \cos \alpha$$

Thus  $n_g = n_l \tan \alpha$

ii)



$$n_a \sin i_1 = n_g \sin i_g$$

$$n_a \sin(\alpha + 8) = n_g \sin\{90 - (\alpha + 8)\}$$

$$n_g = \frac{\sin(\alpha + 8)}{\cos(\alpha + 8)} = \tan(\alpha + 8)$$

$$\text{but } n_g = \frac{\frac{n_g}{n_l} + \tan 8}{1 - \frac{n_g}{n_l} \tan 8}$$

$$n_g = \frac{n_g + n_l \tan 8}{n_l - n_g \tan 8}$$

$$= \frac{n_g + 1.33 \tan 8}{1.33 - n_g \tan 8}$$

$$0.140 n_g^2 - 0.33 n_g + 0.1869 = 0$$

$$n_g = 1.39 \text{ or } n_g = 0.95$$

## APPLICATIONS OF TOTAL INTERNAL REFLECTION.

1. The mirage. A mirage is a shimmering or shinning surface. During hot days, air is progressively hotter towards the ground. The density of air therefore decreases as one moves downwards and thus light travels progressively from denser to less dense layers of air. The light rays are thus reflected away from the normal until the ray suffers total internal reflection when the critical angle is exceeded.

An observer thus gets an impression of a pool of water on the ground in front specially on a smooth tarmac road. This apparent shimmering pool of water is the mirage.

2. Radio waves. A radio wave is electromagnetic in nature since it has electric and magnetic forces in it. Light waves are also electro-magnetic and so they are similar. When radio waves are sent from a transmitting station on the earth, the waves soon enter a region of high density of electrons called an Appleton layer well above the earth. The radio waves are then refracted away from the normal in this layer. At some height or altitude, total internal reflection occurs and the waves begin to move downwards towards a receiving station on earth, i. e

3. Air cell method for determining refractive index of a liquid. Two thin plane parallel glass plates are connected together so as to contain a thin film of air between them. This forms an air cell is placed inside the liquid i.e.

A bright source,  $S$  of light sends light in the direction  $SO$  from one side of  $X$  and observation is done from the other side at the eye point  $E$ .

For light ray incident normally at  $X$ , there is no reflection as the light passes straight through the cell as observed from  $E$ . the air cell is then rotated about its vertical axis until the light is cut off from  $E$ . Total internal reflection will take place.

The ray is refracted away along  $OB$  in the glass and at  $B$  total internal reflection occurs. Let;

$i_1$  = angle of incidence in the liquid.

$i_2$  = angle of refraction in glass.

$n_l$  = refractive index of the liquid.

$n_g$  = refractive index of glass.

For parallel boundaries,